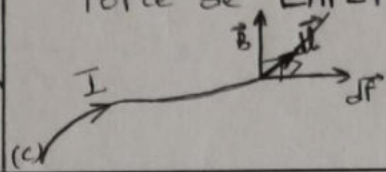
Electromagnétisme :

→ Force électrostatique : $\vec{F_e} = q\vec{E}$

→ Force magnétique : $\vec{F_m} = q\vec{V} \wedge \vec{B} = q\vec{E_m}$

$\vec{V} \wedge \vec{B}$ = champ électromoteur

Force de LAPLACE :

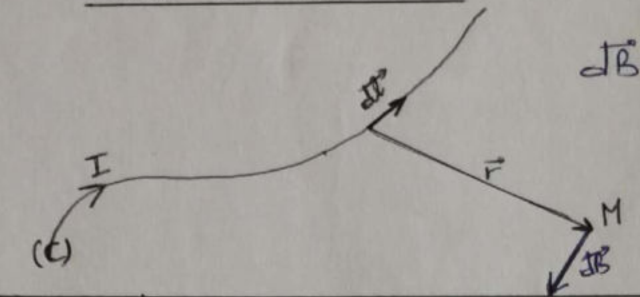


$$\vec{dF} = I\,\vec{dl} \wedge \vec{B}$$

Force de Lorentz :

$$\vec{F} = q\,(\vec{E} + \vec{V} \wedge \vec{B})$$

Loi de Biot & Savart :



$$\vec{dB} = \frac{\mu_0 I}{4\pi} \frac{\vec{dl} \wedge \vec{r}}{r^3}$$

$\mu_0 = 4\pi . 10^{-7}$ H/m

Théorème d'Ampère :

$$\mathcal{C} = \oint_{(C)} \vec{B} . \vec{dl} = \mu_0 \sum_j I_j$$

le théoreme n'est valable que ds le vide. On peut cependant l'appliquer ds l'air.

→ **Théoréme de Green-Ostrogradsky :**

$$\iiint_V div\vec{V}\, dV = \oiint_S \vec{V} . \vec{dS}$$

→ **théorème de Stokes :**

$$\iint_S (\vec{rot}\ \vec{V})\, \vec{dS} = \oint_C \vec{V} . \vec{dl}$$

Equations Locales :

→ en electrostatique : $\begin{cases} - \vec{rot}\, E = 0 \\ - div\, E = \frac{\rho}{\varepsilon} \end{cases}$

→ en Magnétostatique : $\begin{cases} - div\, B = 0 \\ - rot\, B = \mu_0 j \end{cases}$

Force électromotrice :

$$e = \oint_C \frac{F(M,t)}{q} \vec{dr}$$

si le contour est en fait un conducteur materiel de résistance R. le courant créé s'exprime par :

$$e(t) = RI$$

Loi de Faraday :

$$e = \frac{-d\Phi}{dt}$$

$\frac{d\Phi}{dt} > 0 \Rightarrow I_{ind}$ crée un champ $\vec{B'}$ de sens ≠ $\vec{B}$

$\frac{d\Phi}{dt} < 0 \Rightarrow I_{ind}$ a le même sens que I qui a créé $\vec{B}$

$$e = \oint_C \vec{E}_{mi}\, \vec{dr}$$

$$\vec{E}_{mi} = -\frac{\partial \vec{A}}{\partial t} + \vec{V} \wedge \vec{B}$$

. B uniforme $\Rightarrow \frac{\partial \vec{A}}{\partial t} = 0$ c.à.d $\vec{E}_{mi} = \vec{V} \wedge \vec{B}$

. Surface fixe $\Rightarrow \vec{V} \wedge \vec{B} = 0$ c.à.d $\vec{E}_{mi} = \frac{-\partial \vec{A}}{\partial t}$

inductance mutuelle :

Pr deux Circuits filiformes C_1 et C_2

$$M = \frac{\Phi_{12}}{I_2} = \frac{\Phi_{21}}{I_1}$$

(Signe dépendant des orientations)

Ebatou Hamza!

Notions Mathématiques

Opérateurs vectoriels

Gradient : $\overrightarrow{\mathrm{grad}} f = \overrightarrow{\nabla} f = \dfrac{\partial f}{\partial x}\vec{e}_x + \dfrac{\partial f}{\partial y}\vec{e}_y + \dfrac{\partial f}{\partial z}\vec{e}_z$

Le gradient est un vecteur qui pointe vers les valeurs croissantes de f. *Rappel* : $\boxed{\mathrm{d}f = \overrightarrow{\nabla} f \cdot \mathrm{d}\vec{r}}$

Divergence : $\mathrm{div}\,\vec{v} = \overrightarrow{\nabla} \cdot \vec{v} = \dfrac{\partial v_x}{\partial x} + \dfrac{\partial v_y}{\partial y} + \dfrac{\partial v_z}{\partial z}$

Formule de Green-Ostrogradsky :

$$\boxed{\Phi = \iint_{S \text{ fermée}} \vec{v} \cdot \overrightarrow{\mathrm{d}S} = \iiint_{\tau} \overrightarrow{\nabla} \cdot \vec{v}\, \mathrm{d}\tau} \qquad (3)$$

Rotationnel : $\overrightarrow{\mathrm{rot}}\vec{v} = \overrightarrow{\nabla} \wedge \vec{v} = \left(\dfrac{\partial v_z}{\partial y} - \dfrac{\partial v_y}{\partial z}\right)\vec{e}_x + \left(\dfrac{\partial v_x}{\partial z} - \dfrac{\partial v_z}{\partial x}\right)\vec{e}_y + \left(\dfrac{\partial v_y}{\partial x} - \dfrac{\partial v_x}{\partial y}\right)\vec{e}_z$

Formule de Stokes :

$$\boxed{\mathcal{C} = \oint \vec{v} \cdot \overrightarrow{\mathrm{d}l} = \iint_S (\overrightarrow{\nabla} \wedge \vec{v}) \cdot \overrightarrow{\mathrm{d}S}} \qquad (4)$$

Le rotationnel mesure si un champ tourne localement.

Laplacien : $\Delta = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$

L'opérateur Laplacien peut s'appliquer à une fonction scalaire

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

ou à un vecteur

$$\Delta \vec{v} = \frac{\partial^2 \vec{v}}{\partial x^2} + \frac{\partial^2 \vec{v}}{\partial y^2} + \frac{\partial^2 \vec{v}}{\partial z^2}.$$

Quelques relations vectorielles

$$\vec{A} \cdot (\vec{B} \wedge \vec{C}) = \vec{B} \cdot (\vec{C} \wedge \vec{A}) = \vec{C} \cdot (\vec{A} \wedge \vec{B})$$
$$\vec{A} \wedge \vec{B} \wedge \vec{C} = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$
$$\mathrm{div}(\overrightarrow{\nabla} f) = \Delta f$$
$$\mathrm{div}(\overrightarrow{\mathrm{rot}}\,\vec{v}) = 0$$
$$\mathrm{rot}(\overrightarrow{\nabla} \mathrm{f}) = 0$$
$$\overrightarrow{\mathrm{rot}}(\overrightarrow{\mathrm{rot}}\,\vec{v}) = \overrightarrow{\nabla}(\mathrm{div}\,\vec{v}) - \Delta \vec{v}$$

Forme explicite des opérateurs vectoriels

- Coordonnées cartesiennes

$$
\begin{aligned}
\vec{\nabla} f &= \frac{\partial f}{\partial x}\vec{e}_x + \frac{\partial f}{\partial y}\vec{e}_y + \frac{\partial f}{\partial z}\vec{e}_z \\
\vec{\nabla}\cdot\vec{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \\
\vec{\nabla}\wedge\vec{v} &= \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\vec{e}_x + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\vec{e}_y + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\vec{e}_z \\
\Delta f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \\
\Delta \vec{v} &= \frac{\partial^2 \vec{v}}{\partial x^2} + \frac{\partial^2 \vec{v}}{\partial y^2} + \frac{\partial^2 \vec{v}}{\partial z^2}
\end{aligned}
$$

Coordonnées cylindriques

$$
\begin{aligned}
\vec{\nabla} f &= \frac{\partial f}{\partial \rho}\vec{e}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{\partial f}{\partial z}\vec{e}_z \\
\vec{\nabla}\cdot\vec{v} &= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho v_\rho) + \frac{1}{\rho}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \\
\vec{\nabla}\wedge\vec{v} &= \left(\frac{1}{\rho}\frac{\partial v_z}{\partial \theta} - \frac{\partial v_\theta}{\partial z}\right)\vec{e}_\rho + \left(\frac{\partial v_\rho}{\partial z} - \frac{\partial v_z}{\partial \rho}\right)\vec{e}_\theta + \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}(\rho v_\theta) - \frac{\partial v_\rho}{\partial \theta}\right)\vec{e}_z \\
\Delta f &= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2} \\
\Delta \vec{v} &= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial \vec{v}}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 \vec{v}}{\partial \theta^2} + \frac{\partial^2 \vec{v}}{\partial z^2}
\end{aligned}
$$

- Coordonnées sphériques

$$
\begin{aligned}
\vec{\nabla} f &= \frac{\partial f}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\vec{e}_\phi \\
\vec{\nabla}\cdot\vec{v} &= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} \\
\vec{\nabla}\wedge\vec{v} &= \left[\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right)\right]\vec{e}_r + \left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{1}{r}\frac{\partial}{\partial r}(r v_\phi)\right]\vec{e}_\theta \\
&+ \frac{1}{r}\left(\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right)\vec{e}_\phi \\
\Delta f &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin\theta}\frac{\partial^2 f}{\partial \phi^2} \\
\Delta \vec{v} &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \vec{v}}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial \vec{v}}{\partial \theta}\right) + \frac{1}{r^2\sin\theta}\frac{\partial^2 \vec{v}}{\partial \phi^2}
\end{aligned}
$$

On remarque que $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r f)$.